CCFU Proof 15

One-Step Lorentz Factor $\gamma_1 = \sqrt{5}/2$

Given. Let $\varphi = (1+\sqrt{5})/2$, so $\varphi^2 = \varphi + 1$. The C_2 one-step rapidity is $\eta = \ln \varphi$.

Part A — Lorentz factor.

$$\gamma_1 = \cosh(\ln \varphi) = \frac{\varphi + 1/\varphi}{2}.$$

Since $1/\varphi = \varphi - 1$:

$$\varphi + 1/\varphi = 2\varphi - 1.$$

Squaring:

$$(2\varphi - 1)^2 = 4\varphi^2 - 4\varphi + 1 = 4(\varphi + 1) - 4\varphi + 1 = 5.$$

Since $2\varphi - 1 > 0$: $2\varphi - 1 = \sqrt{5}$. Therefore:

$$\gamma_1 = \frac{\sqrt{5}}{2}. \qquad \blacksquare$$

Part B — Reciprocal of 2φ.

$$\frac{1}{2\varphi} = \frac{1}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{\sqrt{5}-1}{4} = 0.30902\ldots \qquad \blacksquare$$